Section 001
CDS 101 *Introduction to Computational and Data Sciences*

Class 26: Modeling II

November 29, 2017

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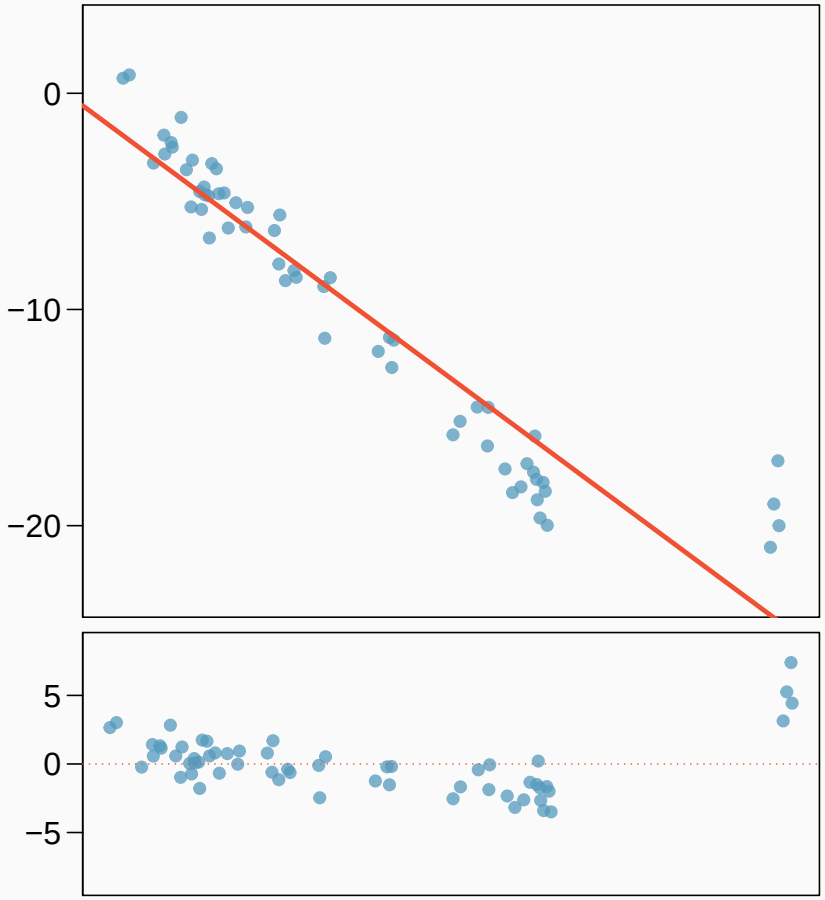
Types of outliers in linear regression

Types of outliers

How do outliers influence the least squares line in this plot?

To answer this question think of where the regression line would be with and without the outlier(s). Without the outliers the regression line would be steeper, and lie closer to the larger group of observations. With the outliers the line is pulled up and away from some of the observations in the larger group.



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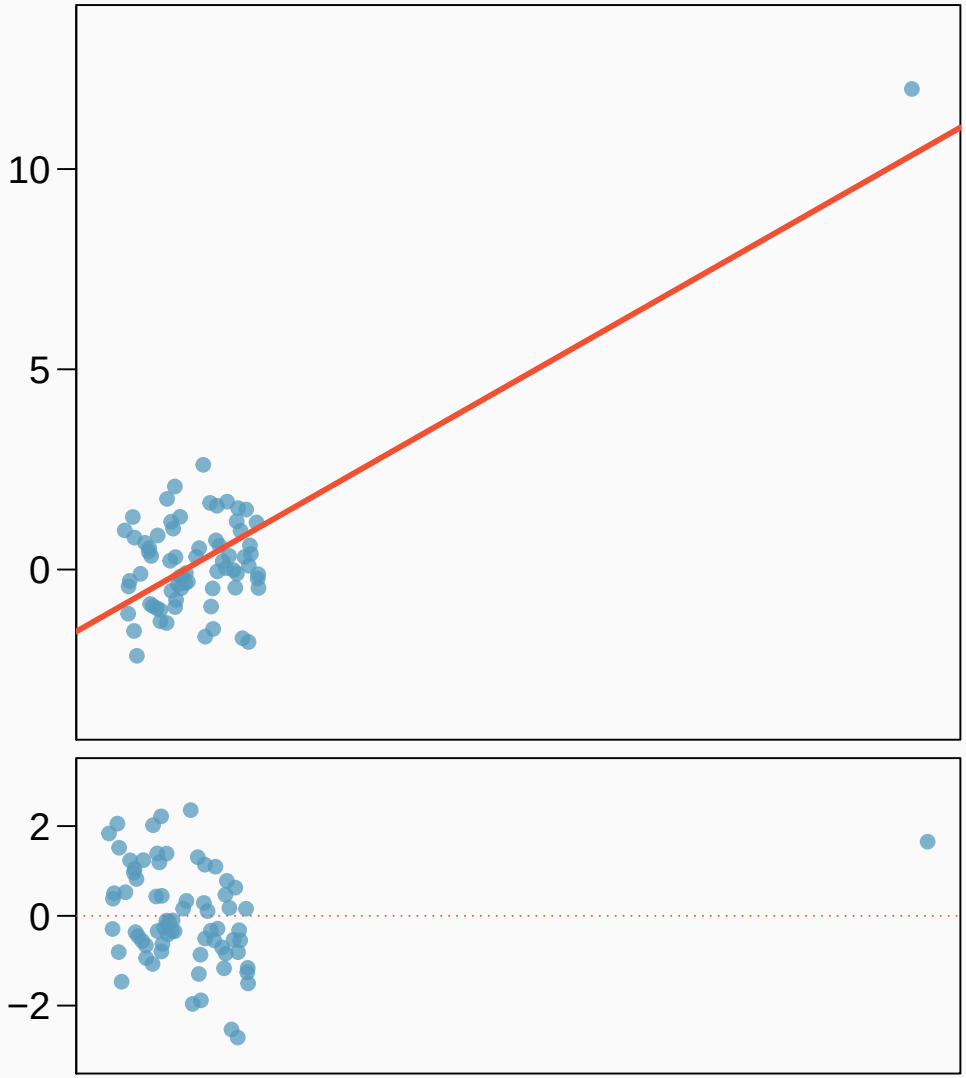

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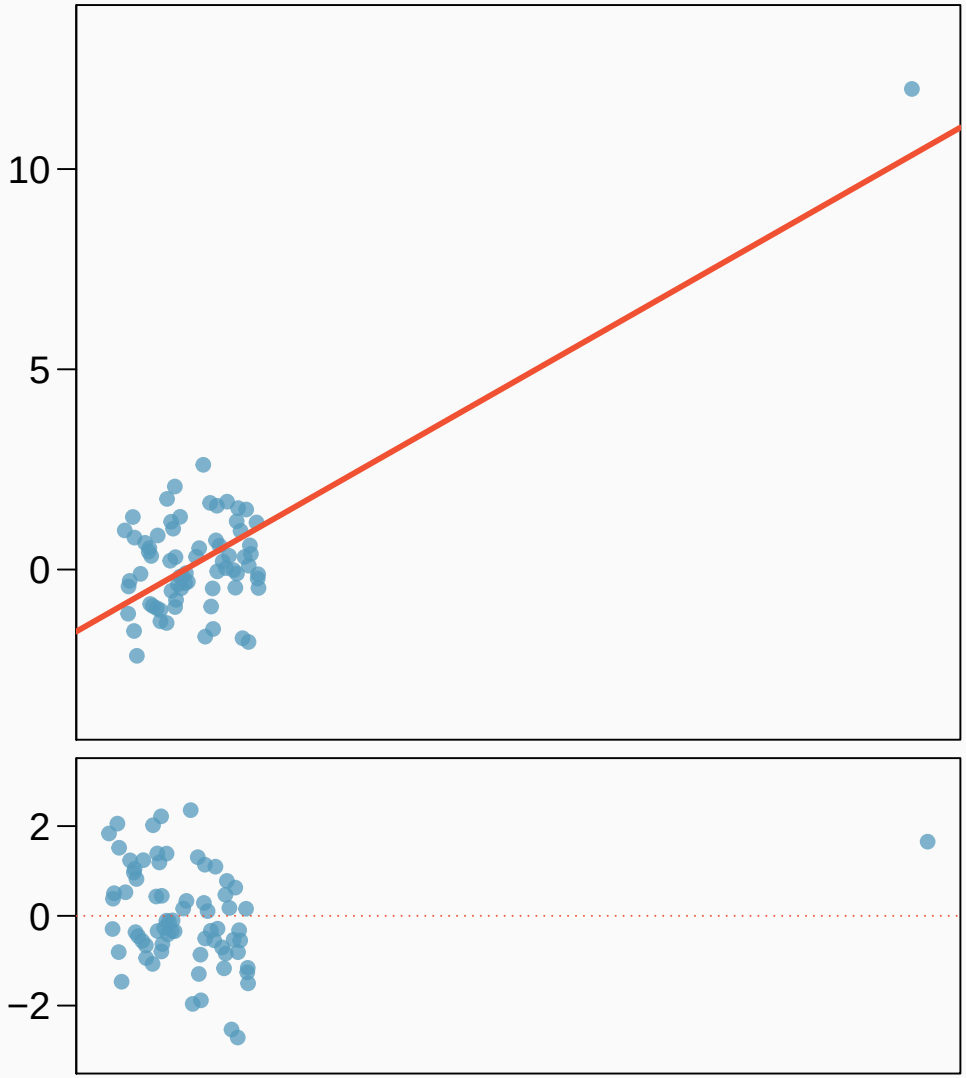
Without the outlier there is no evident relationship between x and y.



Some terminology

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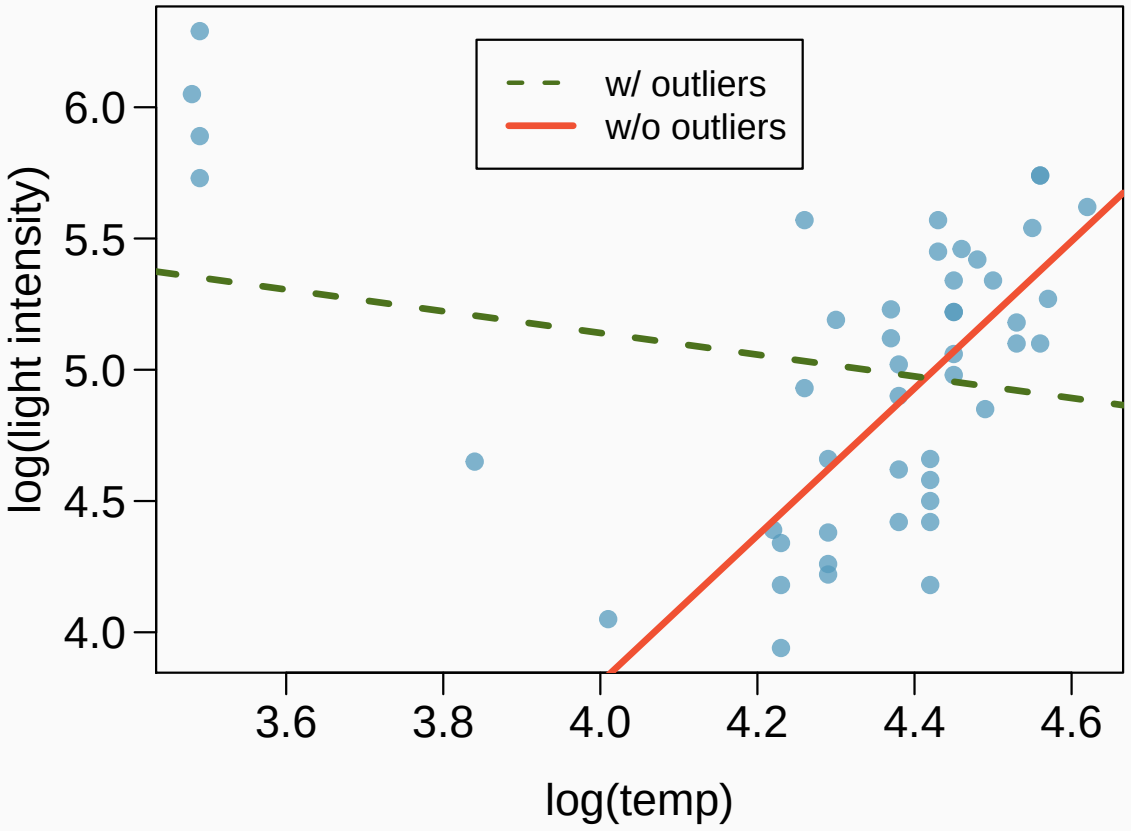
- Outliers that lie horizontally away from the center of the cloud are called *high leverage* points.
- High leverage points that actually influence the slope of the regression line are called *influential* points.
- In order to determine if a point is influential, visualize the regression line with and without the point. Does the slope of the line change considerably? If so, then the point is influential. If not, then itÕs not an influential point.

Influential points

Data are available on the log of the surface temperature and the log of the light intensity of 47 stars in the star cluster CYG OB1.



Types of outliers

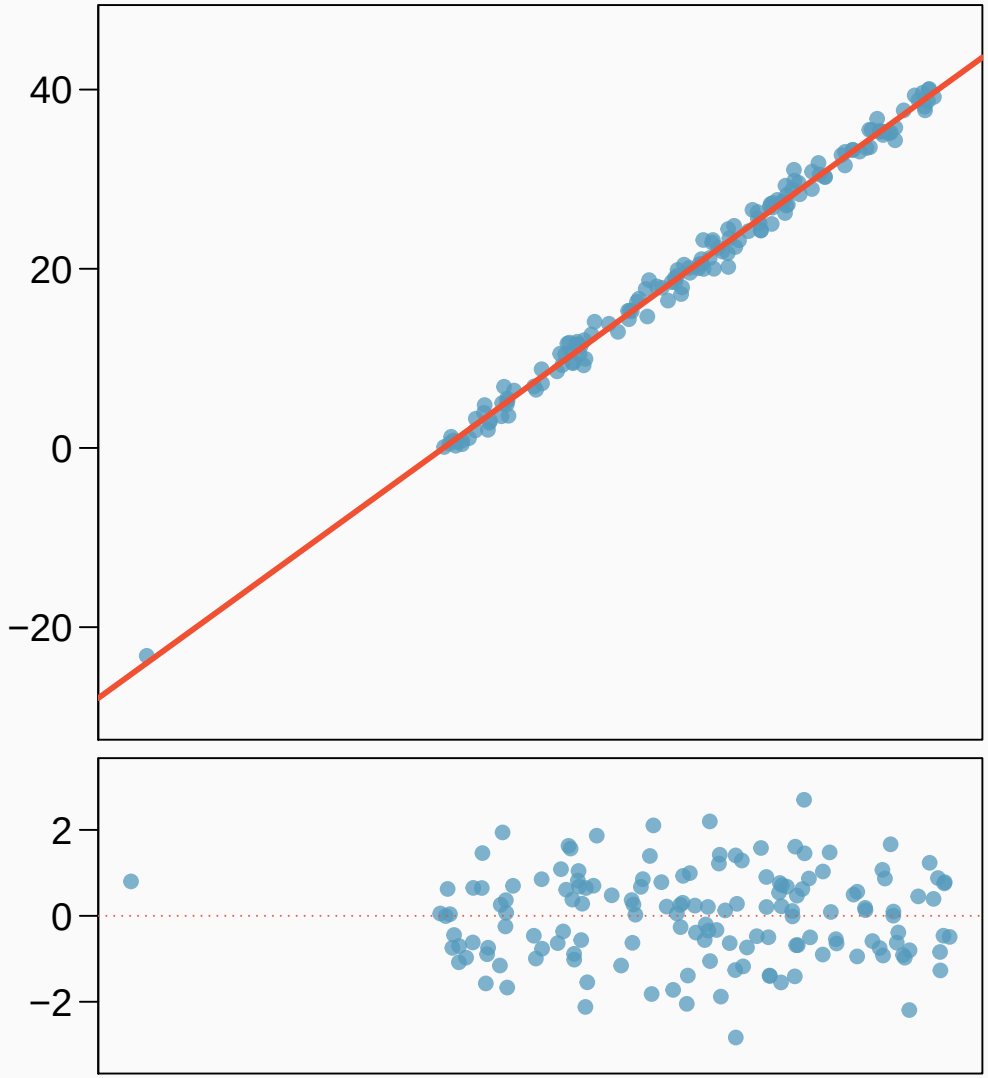
Which of the below best describes the outlier?

(a) influential

(b) high leverage

(c) none of the above

(d) there are no outliers



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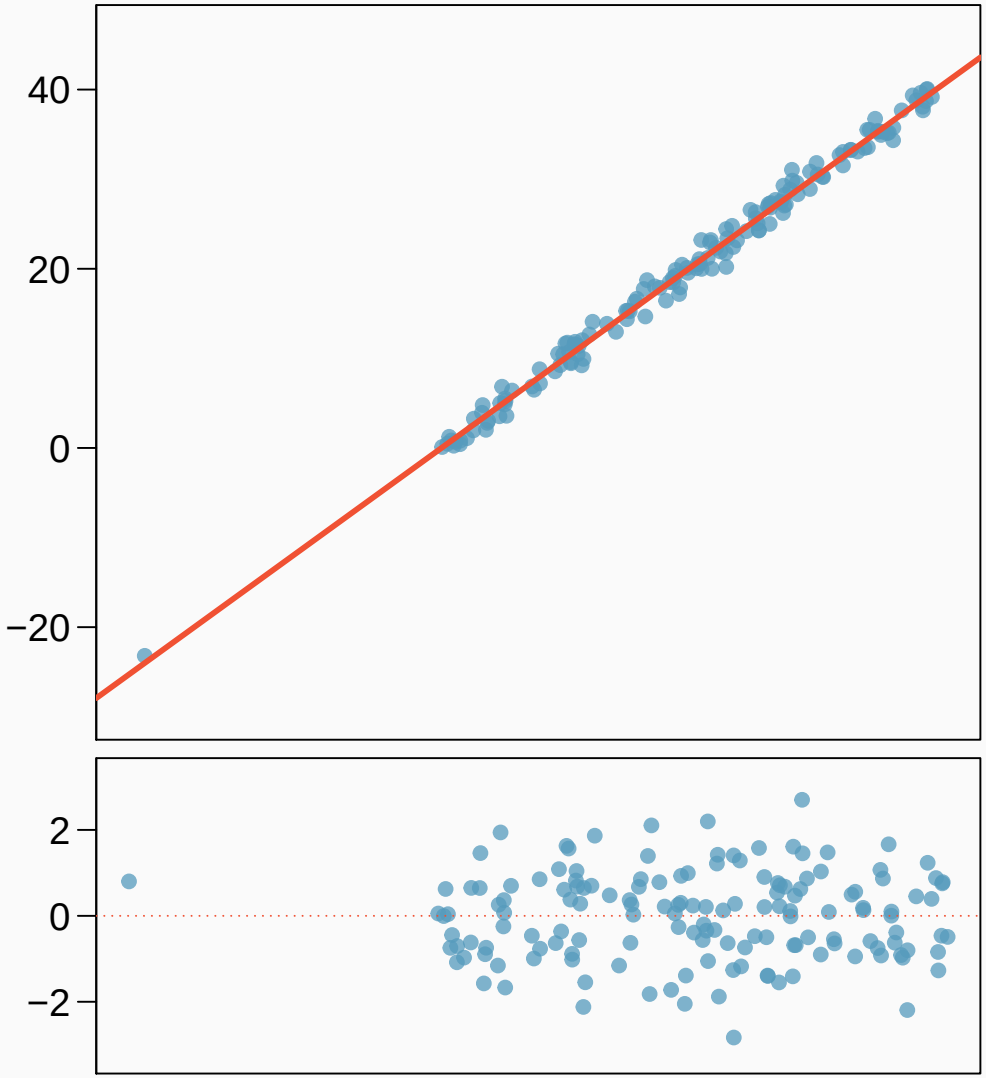

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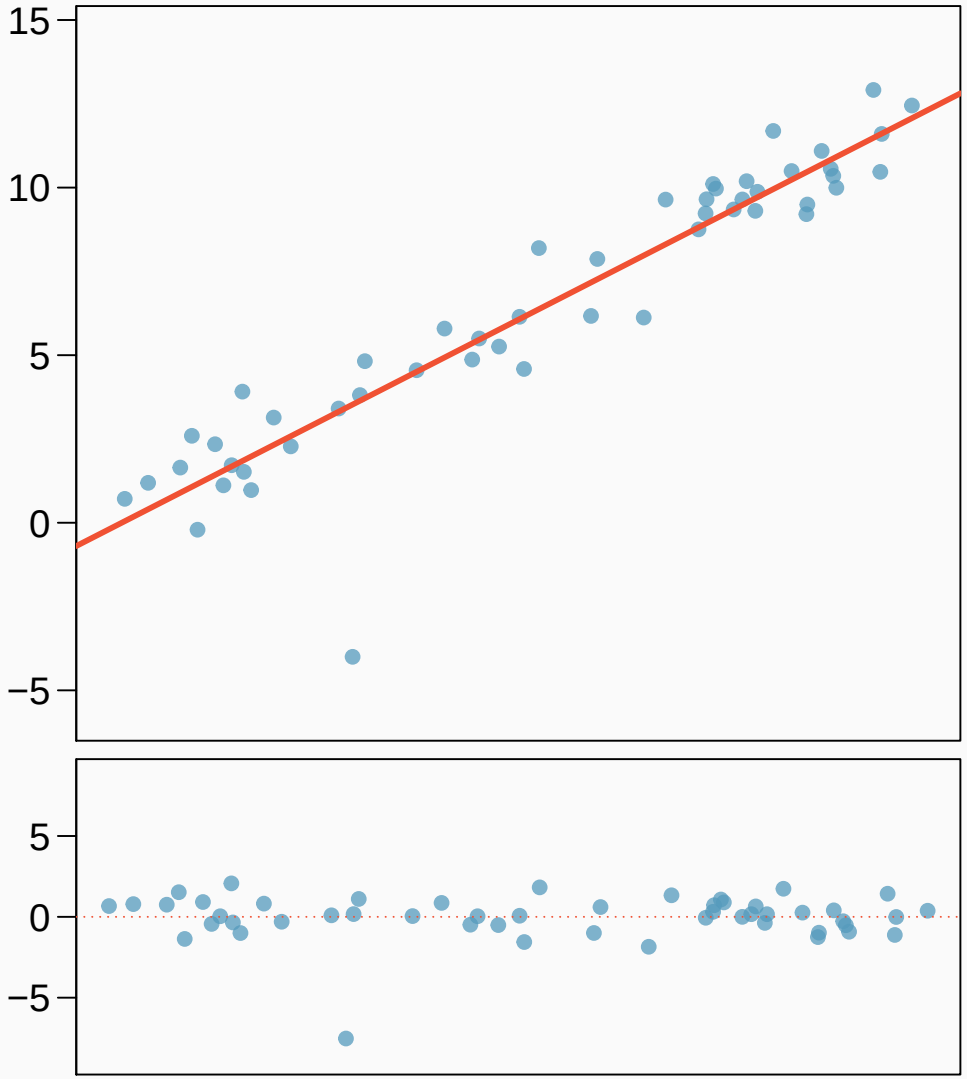
Does this outlier influence the slope of the regression line?



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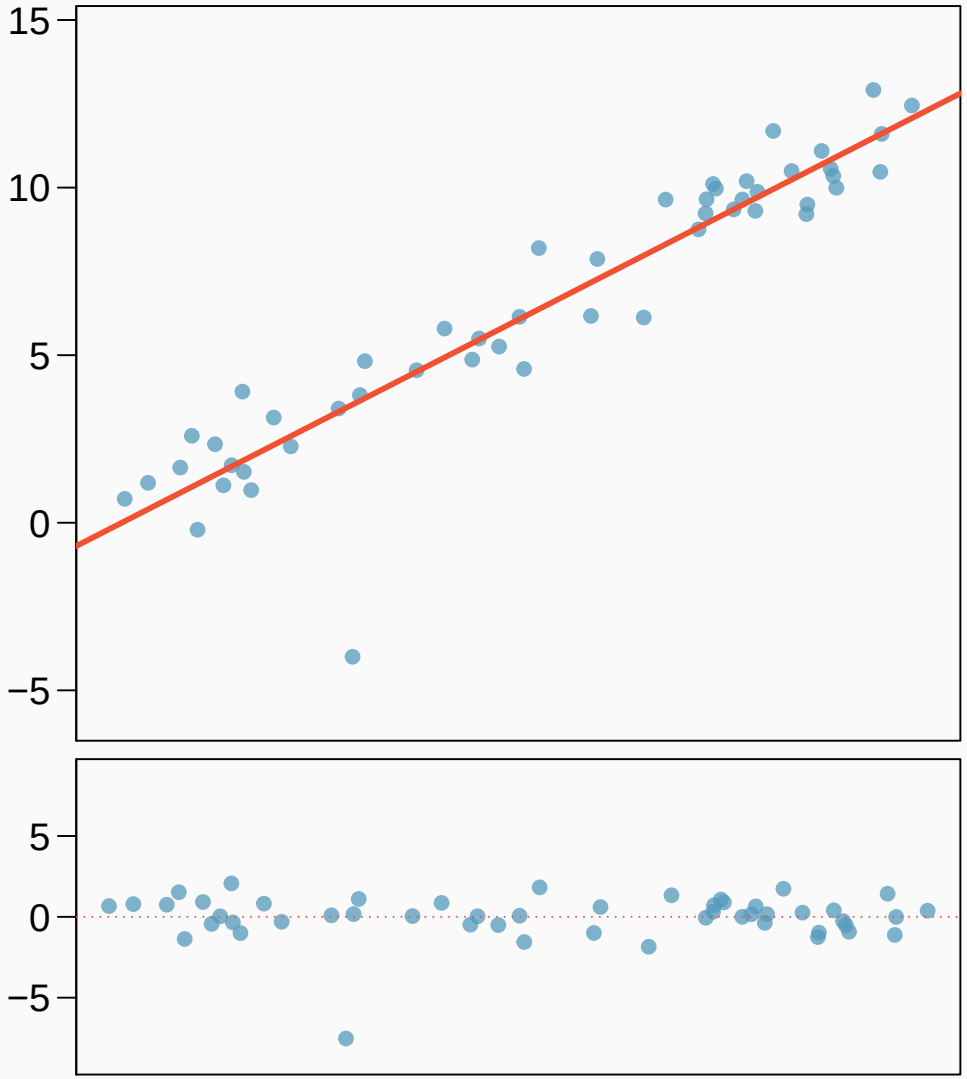
Does this outlier influence the slope of the regression line?

Not much...



Recap

Which of following is true?

(a) Influential points always change the intercept of the regression line.

(b) Influential points always reduce R^2.

(c) It is much more likely for a low leverage point to be influential, than a high leverage point.

(d) When the data set includes an influential point, the relationship between the explanatory variable and the response variable is always nonlinear.

(e) None of the above.

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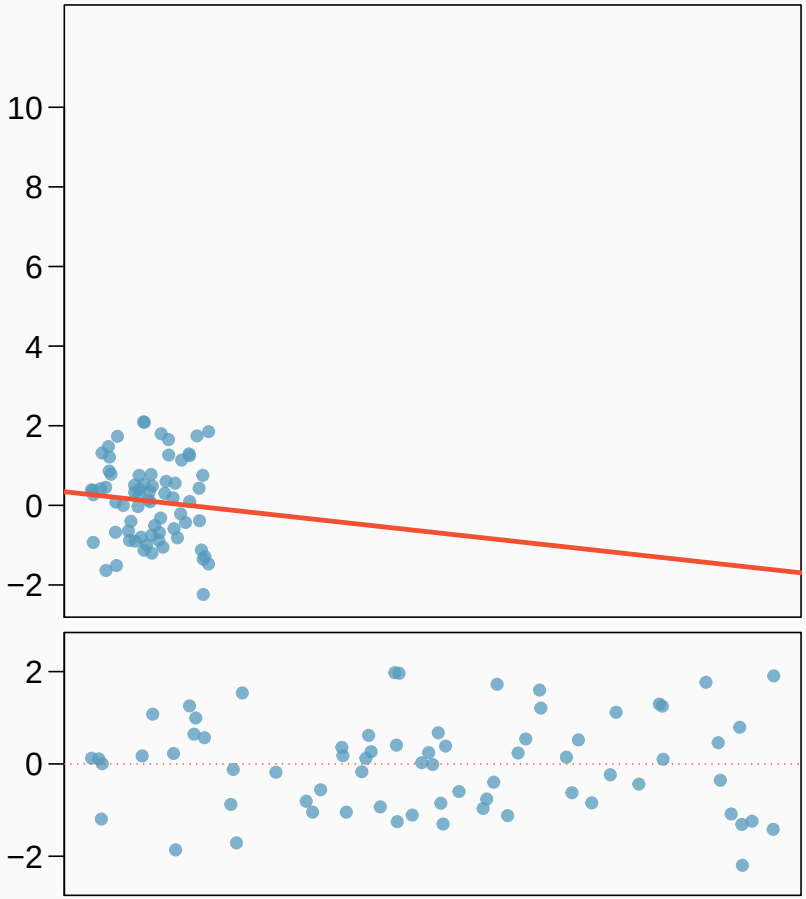
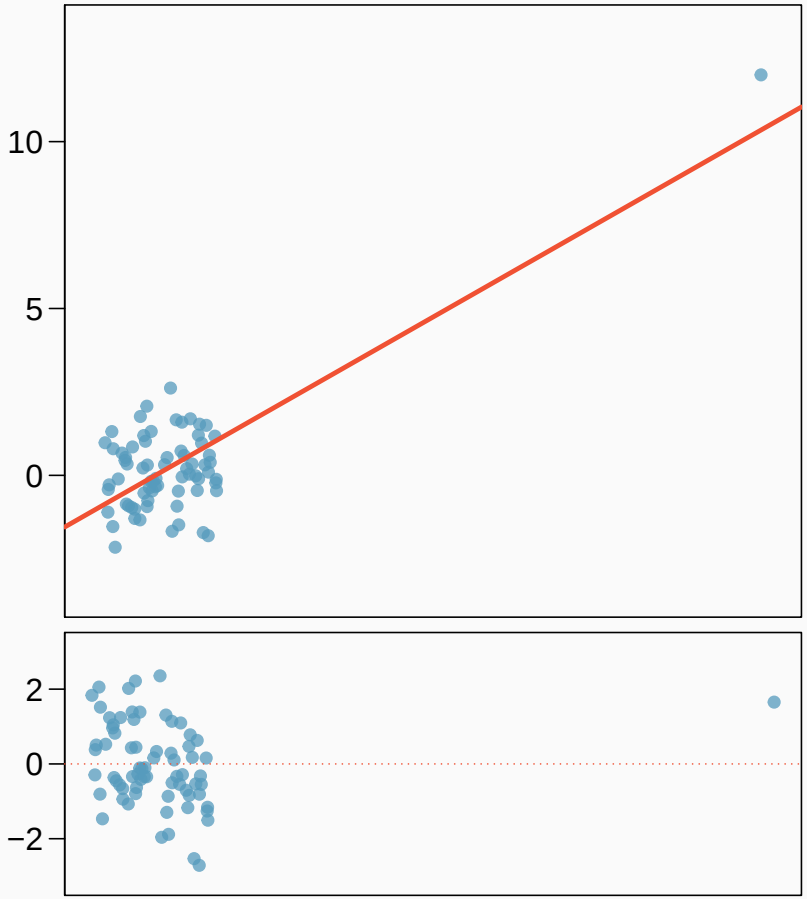
Recap (cont.)

$R = 0.08, R^2 = 0.0064$



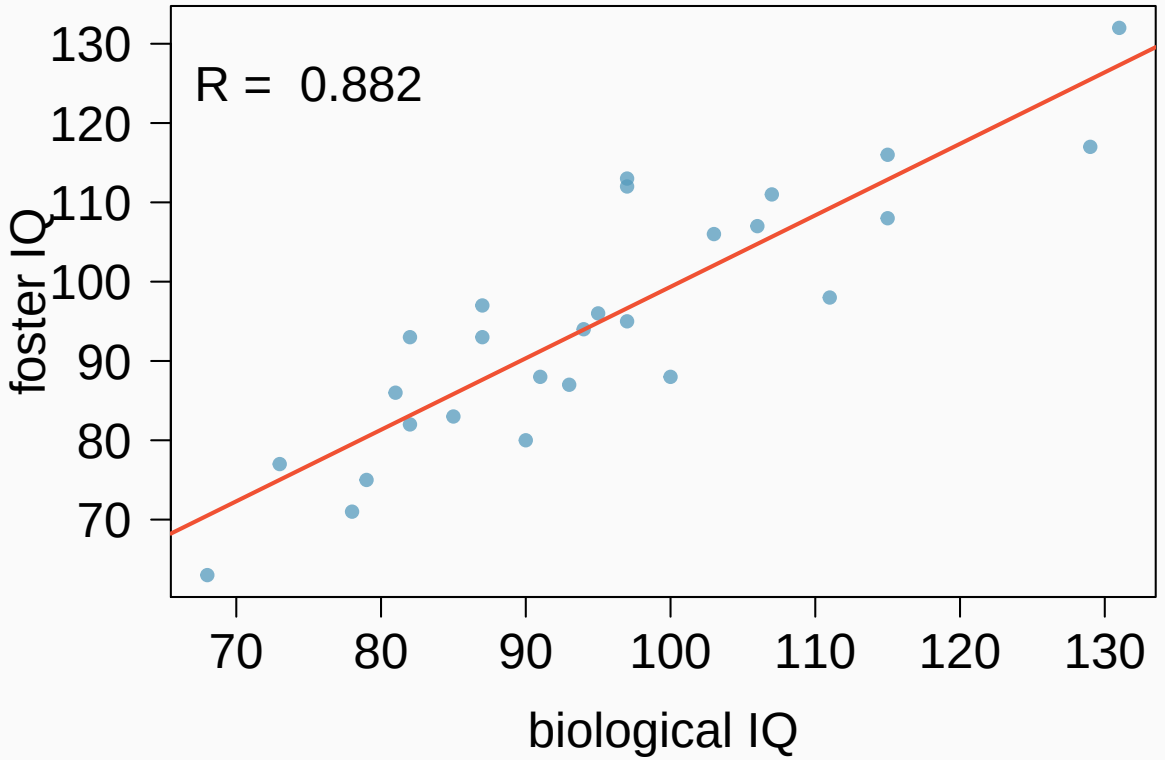
$R = 0.79, R^2 = 0.6241$



Inference for linear regression

Nature or nurture?

In 1966 Cyril Burt published a paper called “The genetic determination of differences in intelligence: A study of monozygotic twins reared apart?” The data consist of IQ scores for [an assumed random sample of] 27 identical twins, one raised by foster parents, the other by the biological parents.



Which of the following is false?

```
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)      9.20760    9.29990   0.990    0.332
bioIQ            0.90144    0.09633   9.358  1.2e-09

Residual standard error: 7.729 on 25 degrees of freedom
Multiple R-squared: 0.7779,Adjusted R-squared: 0.769
F-statistic: 87.56 on 1 and 25 DF,  p-value: 1.204e-09
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(a) Additional 10 points in the biological twin's IQ is associated with additional 9 points in the foster twin's IQ, on average.

(b) Roughly 78% of the foster twins' IQs can be accurately predicted by the model.

(c) The linear model is $\widehat{fosterIQ} = 9.2 + 0.9 \times bioIQ$.

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Testing for the slope

Assuming that these 27 twins comprise a representative sample of all twins separated at birth, we would like to test if these data provide convincing evidence that the IQ of the biological twin is a significant predictor of IQ of the foster twin. What are the appropriate hypotheses?

(a) $H_0 : b_0 = 0; H_A : b_0 \neq 0$

(b) $H_0 : \beta_0 = 0; H_A : \beta_0 \neq 0$

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Testing for the slope (cont.)

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  Remember: *We lose 1 degree of freedom for each parameter we estimate, and in simple linear regression we estimate 2 parameters,* β_0 *and* β_1.

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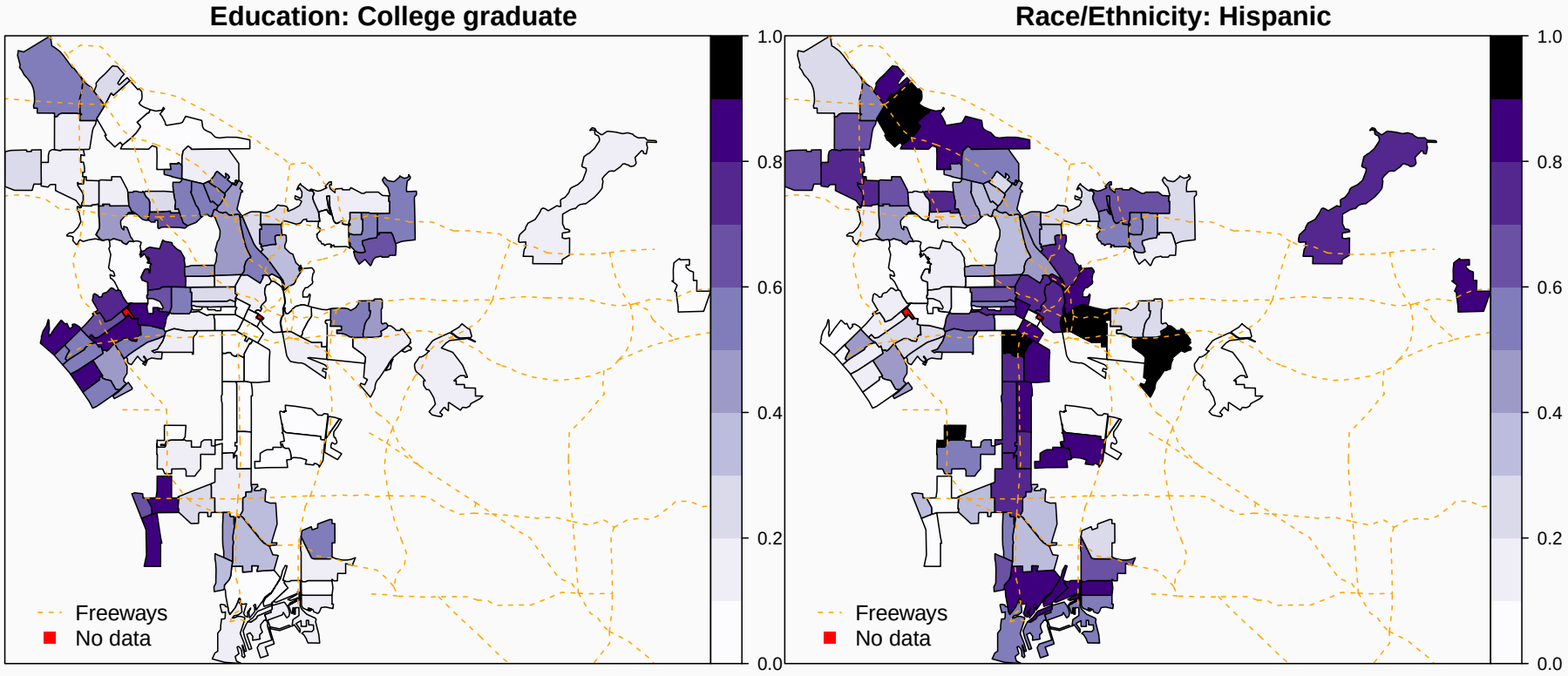
$$
\begin{aligned}
T &= \frac{0.9014 - 0}{0.0963} = 9.36 \\
df &= 27 - 2 = 25 \\
p - value &= P(|T| > 9.36) < 0.01
\end{aligned}
$$

% College graduate vs. % Hispanic in LA

What can you say about the relationship between % college graduate and % Hispanic in a sample of 100 zip code areas in LA?



% College educated vs. % Hispanic in LA - another look

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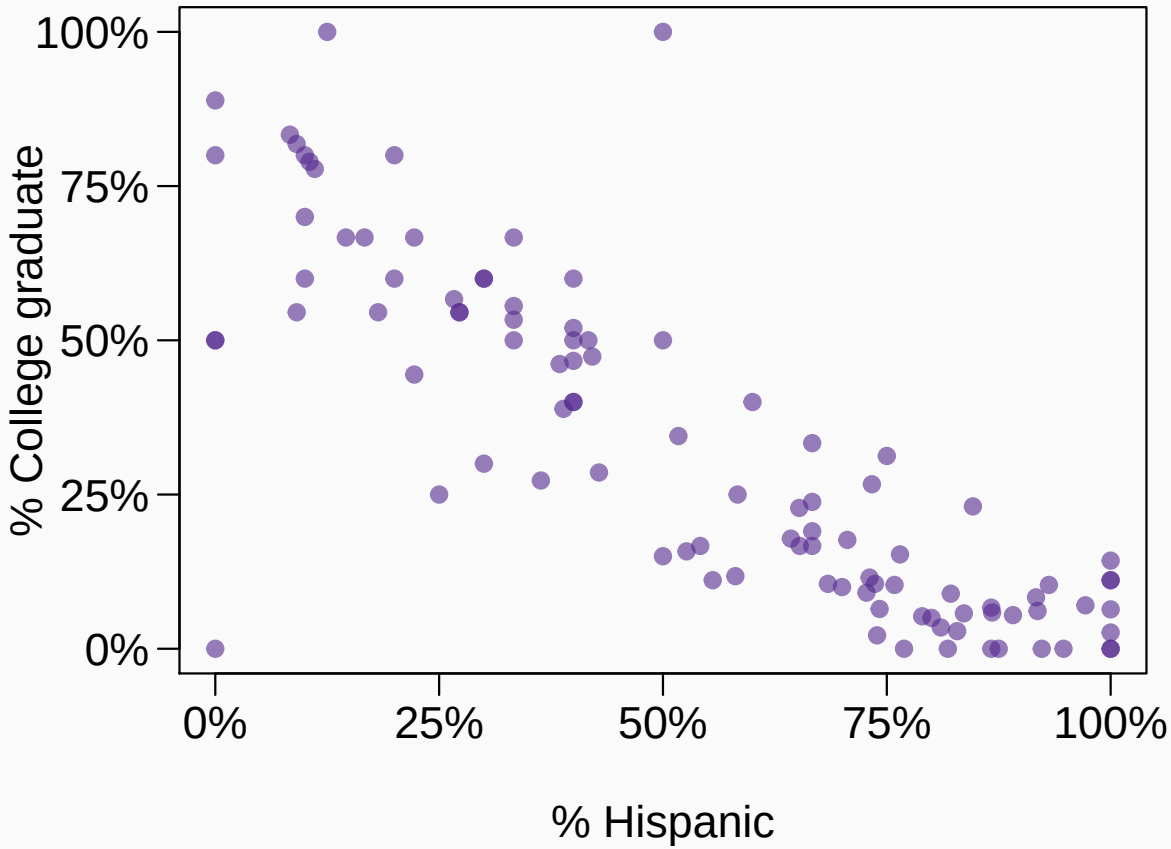

% College educated vs. % Hispanic in LA - linear model

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(Intercept)	0.7290	0.0308	23.68	0.0000
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(a) A 1% increase in Hispanic residents in a zip code area in LA is associated with a 75% decrease in % of college grads.

(b) A 1% increase in Hispanic residents in a zip code area in LA is associated with a 0.75% decrease in % of college grads.

(c) An additional 1% of Hispanic residents decreases the % of college graduates in a zip code area in LA by 0.75%.

(d) In zip code areas with no Hispanic residents, % of college graduates is expected to be 75%.

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Not very...

Confidence interval for the slope

Remember that a confidence interval is calculated as $point\ estimate \pm ME$ and the degrees of freedom associated with the slope in a simple linear regression is $n-2$. Which of the below is the correct 95% confidence interval for the slope parameter? Note that the model is based on observations from 27 twins.

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$$0.9014 \pm 2.06 \times 0.0963$$

$$(0.7 \ , \ 1.1)$$

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- The regression output gives b_1, SE_{b_1}, and *two-tailed* p-value for the t-test for the slope where the null value is 0.
- We rarely do inference on the intercept, so we'll be focusing on the estimates and inference for the slope.

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- If you have a sample that is non-random (biased), inference on the results will be unreliable.

Caution

- Always be aware of the type of data you're working with: random sample, non-random sample, or population.
- Statistical inference, and the resulting p-values, are meaningless when you already have population data.
- If you have a sample that is non-random (biased), inference on the results will be unreliable.
- The ultimate goal is to have independent observations.